Frege's Puzzle Again

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Outline

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Frege's Puzzle

- $1 + 6 = 7$.
- The morning star is identical to the evening star.
- Cicero is Tully.
- Beihai Zhou is Yu's advisor.

Frege's Puzzle

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Frege believed that these statements all have the form 'a=b', where 'a' and 'b' are either names or descriptions that *denote* individuals.

Naturally, 'Cicero is Tully' is true iff. the person Cicero just is the person Tully. Frege (1892) noticed that 'a=a' has a cognitive significance that must be different from the cognitive significance of 'a=b'.

We can learn that 'Cicero = Cicero' is true simply by inspecting it;
as for 'Cicero = Tully', you have to examine the world to see whether the two persons are the same.

Frege's Puzzle

So the puzzle Frege discovered is: how do we account for the difference in cognitive significance between 'a=b' and 'a=a' when they are true?

The general Frege's Puzzle also could contain:

- attitude ascription version
- the version without mention identity symbol and belief/doxastic state.
  1. If Hesperus is a planet, Hesperus is.
  2. If Hesperus is a planet, Phosphorus is.

Frege's solution: meaning consists of sense and reference.

Frege's Puzzle

Frege's intention (周北海, 2010):

- Is equality is a relation? A relation between objects, or between names or signs of objects?
- **Begriffsschrift** (1879):
  - symbols represent their content, but"they at once stand for themselves as soon as they are combined by the symbol for identity of content". (Beaney, 1997)
  - (A=B) is therefore to mean: "the symbol A and the symbol B have the same conceptual content, so that A can always be replaced by B and *vice versa*".
- **Über begriff und gegenstand** (1891): conceptual content consists of sense and reference, the symbol A and the symbol B have the same reference.
- **Über sinn und bedeutung** (1892): identity is a relation of senses.

Frege's Puzzle

Frege's Puzzle doesn't include:

- In the case a=b only concerns with "mode of designation", "the cognitive value of a=a becomes essentially equal to that of a=b, provided a=b is true." "A difference can arise only if the difference between the signs corresponds to a difference in the mode of presentation of the thing designated." (*Über sinn und bedeutung, 1892*)
- Salmon (1986) finds Frege himself was aware of the distinction: expressed thought and thought the speaker leads others to take as true although he doesn't express them. (Frege, 1897)
- "We are to suppose that the audience has complete mastery of both items and finds the utterance or inscription informative nevertheless."(Salmon, 1986)

Frege's Puzzle

Frege's puzzle of identity statements, in its simplest form, can be stated as the following question (Wang and Fan, 2015):

How do we explain the difference between $a = a$ and $a = b$ in cognitive value to a linguistically competent speaker when a and b are co-referential?

1. What is the concept of 'cognitive value'?
2. What is the concept of 'linguistic competence'?
3. What is the proposition expressed by $a = b$ exactly?

Outline

The Antinomy of the Variable

Bertrand Russell:

> *"x is, in some sense, the object denoted by any term; yet this can hardly be strictly maintained, for different variables may occur in a proposition, yet the object denoted by any term, one would suppose, is unique." (Russell, 1903)*

Kit Fine(2003) and (2007) brought the problem back to public attention.

> *"Once Gottlob Frege had provided a clear syntactic account of variables and once Alfred Tarski had supplemented this with a rigorous semantic account, it would appear that there was nothing more of any significance to be said. It seems to me, however, that this common view is mistaken."(Fine, 2003)*

The Antinomy of the Variable

- (1) Every number is less than or equal to itself.
- (2) Every number is less than or equal to some number.

- (1*) $\forall x x \leq x$
- (2*) $\exists y \forall x x \leq y$

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What is the syntactic structure of sentences (1*) and (2*)?

The standard answer since Tarski is . . .

The Antinomy of the Variable

(1) This standard account presupposes assumption

- *(α): Variables are genuine syntactic constituents of quantified sentences.*

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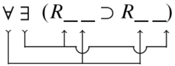
It's non-trivial. Frege rejects assumption (α).



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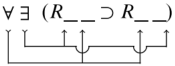

Predicates result from "removing" occurrences of a name from a sentence. We could permit gaping formulas and use wiring diagrams to link the quantifier to its gaps and to channel in values.

The Antinomy of the Variable

(2) Writing quantified sentences using variables resolves ambiguities and facilitates inference because it wears its compositional structure on its sleeve.

(3) Assuming (α), the variable must have some meaning or "semantic role" or "linguistic function".

The Antinomy of the Variable

The antinomy of the variable concerns whether two variables, '*x*' and '*y*', agree in meaning. The difficulty is "we wish to say contradictory things about their semantic role".

- *Sameness*: In sentences that differ in the total, proper substitution of '*x*' for '*y*', these variables have the same meanings.
- *Difference*: When variables '*x*' and '*y*' jointly occur in a single sentence, they have distinct meanings.

We want to show the underlying theoretical motivations for ascribing each feature to variables.

The Antinomy of the Variable

Why 'x' and 'y' Must Not Agree in Meaning ?

- Substituting one for the other may fail to preserve meaning.
- The argument implicitly appeals to the principle of compositionality.
- Compositionality helps explain why speakers can grasp the infinitely many sentences of a language. It also constrains the choice of semantic theories, making them more susceptible to empirical disconfirmation.
- 53213 times 3 equals 159639.

The Antinomy of the Variable

Why '*x*' and '*y*' Must Agree in Meaning ?

- *"Suppose that we have two variables, say 'x' and 'y'. . . .[W]hen we consider their semantic role in two distinct expressions – such as '*$x > 0$*' and '*$y > 0$*', we wish to say that their semantic role is the same. Indeed, this would appear to be as clear a case as any of a mere "conventional" or "notational" difference; the difference is merely in the choice of the symbol and not in its linguistic function." (Fine, 2003)*
- Fine does not elucidate the theoretical importance of this commonality in meanings.

The Antinomy of the Variable

Appealing to a strong notion of *synonymy* within the formal semantics tradition.

This tradition aims at specifying the truth conditions of a sentence in terms of the compositional semantic values of its constituents.(Lewis, 1970; Montague, 1974)
The truth conditions of a sentence will be specified as the set of points of evaluation in which the sentence is true.

The problem: too coarse-grained to serve as the meanings of sentences.

Standard address: *structured meaning*,
(*) John loves Mary. expresses a proposition/meaning consisting of John, the loving relation and Mary, bound together in some way into a unity.
Letting 'j' stand for John, 'm' for Mary and 'L' for the loving relation:
(**) [j[L[m]]]

The Antinomy of the Variable

It traces back to Carnap's strongest notion of synonymy, "intensional isomorphism".

Stalnaker: Meaning of a sentence as "the recipe for determining its truth-conditions as a function of the meanings of its components and the compositional rules."

Lewis: "Differences in intension, we may say, give us coarse differences in meaning . . . For still finer differences in meaning we must look in turn to the intensions of constituents of constituents, and so on. Only when we come to non-compound, lexical constituents can we take sameness of intension as a sufficient condition of synonymy.(Lewis, 1970)

The Antinomy of the Variable

When the antinomy is construed in terms of structured meanings, it derives its force from the conjunction of assumption (α) with an additional assumption (β)

- *(β) Each syntactic constituent of a sentence of a regimented language must correspond to a constituent of the structured meaning of that sentence.*

The Antinomy of the Variable

Structured meanings also have been put to work in developing an account of the information value or belief content of a sentence, which can solve puzzles associated with propositional attitude ascriptions.

Carnap argued that belief ascriptions are neither extensional nor intensional.

Fine's claim that 'x' and 'y' agree in meaning can be bolstered in terms of structured meanings.

One corollary of assumption (β): if Φ and Ψ are synonymous (that is, they have the same structured meaning), then each component α of Φ must agree in meaning–in the relevant sense–with its counterpart β of Ψ.

The Antinomy of the Variable

What's the antinomy of the variables?

We have uncovered that '*x*' and '*y*' must–in some sense–agree in meaning, but also that they must–in some sense–disagree in meaning.

If we want to guarantee that '*x*' and '*y*' have the same meaning, then:

- (3) *Rxx*
- (4) *Rxy*

DIFFERENCE: (3) and (4) differ semantically.
COMPOSITIONALITY: If formulae (3) and (4) differ only by the substitution of constituents which are semantically the same, then (3) and (4) are semantically the same.
MINIMAL PAIR: Formulae (3) and (4) differ only by the substitution of '*x*' for '*y*'–all other inputs to semantic evaluation coincide.
SYNONYMY: '*x*' and '*y*' are semantically the same.

The antinomy is just like Frege's puzzle.

The Antinomy of the Variable

It also bears a close puzzle concerning names, though with variables taking the place of names.

- (5) Hesperus is Hesperus.
- (6) Hesperus is Phosphorus.

(5) and (6) differ semantically, because (6) expresses a valuable extension of our knowledge, while (5) doesn't.

Assuming that the meaning of a name is its referent, and focussing on the simple sentences, the conflict can be brought out as a tension between the following claims:

The Antinomy of the Variable

DIFFERENCE: (1) and (2) differ semantically.

COMPOSITIONALITY: If sentences (1) and (2) differ only by the substitution of constituents which are semantically the same, then (1) and (2) are semantically the same.

MINIMAL PAIR: Sentences (1) and (2) differ only by the substitution of 'Cicero' for 'Tully'–all other inputs to semantic evaluation coincide.

SYNONYMY: 'Cicero' and 'Tully' are semantically the same.

The Antinomy of the Variable

We also have strong theoretical motivations to identity the meaning of co-referential proper names.

- Kripke (1972): Millianism
- Salmon (1986): The Naive Theory (of names)
- Soames (2005): Direct Reference Theory

Frege's puzzle was viewed as a rejection to Milliianism in the tradition of philosophy of language.

> *"Current philosophical thinking on Frege's puzzles has reached an impasse, with strong theoretical arguments in favor of [DIFFERENCE] and strong intuitive arguments in favor of [SYNONYMY] and yet no apparent way to choose between them. And this suggests that we should perhaps take more seriously the possibility of rejecting the assumption of [COMPOSITIONALITY] that puts them in conflict."(Fine, 2007)*

The Antinomy of the Variable

- (3) Rxx
- (4) Rxy

DIFFERENCE: Fine claims the open sentences (3) and (4) differ semantically. Specifically, they embed differently: $\exists x \exists y Rxx$ may be false while $\exists x \exists y Rxy$ is true.
SYNONYMY: Fine suggests that the difference between ‘x’ and ‘y’ is merely “notational”, “It is not as if the variables ‘x’ and ‘y’ have a special ‘x’–sense or ‘y’–sense” (Fine, 2007)

The Antinomy of the Variable

Fine' s idea:

- We must allow that any two variables will be semantically the same, even though pairs of identical and of distinct variables are semantically different;
- be open to the possibility that the meaning of the expressions of a language is to be given in terms of their semantic relationships to one another.

RELATIONISM: The truth conditions of a sentence are not determined by the semantic features of its constituents in isolation, but instead determined by the semantic relationships that hold among the sequence of its constituents as a whole.

The Antinomy of the Variable

Fine semantically evaluates an expression in terms of the semantic connection, $[]$, on its constituents taken in sequence.

Evaluate the sentence '$x + y = y + x$' for truth or falsity in terms of the sequence $< x, +, y, =, y, +, x >$.

The complex expression is said to "give way to" the sequence of its constituents expressions whose semantic connection determines the possible values of the complex expression.

In order to recursively implement this idea, one must define the contribution of an expression χ of arbitrary complexity to the semantic connection on a sequence $(, \chi, \Upsilon)$ that contains χ.

$$\phi = \pi^n \alpha_1 \ldots \alpha_n \mid \neg\phi \mid (\phi \wedge \phi) \mid \forall\alpha\phi$$

The Antinomy of the Variable

TRUTH: ϕ *is true* (in $\mathfrak{A}$) iff $[\![\phi]\!] = \{1\}$
FALSITY: ϕ *is false* (in $\mathfrak{A}$) iff $[\![\phi]\!] = \{0\}$

VARIABLES: $[\![\alpha_1, \ldots, \alpha_n]\!] = \{\langle d_1, \ldots, d_n \rangle \in D^n \mid d_i = d_j \text{ if } \alpha_i = \alpha_j\}$

ATOMIC:

- $(\sigma, 1, \nu) \in [\![\Sigma, \pi\alpha_1 \ldots \alpha_k, \Upsilon]\!]$ iff for some τ such that$(\sigma, \tau, \nu) \in [\![\Sigma, \alpha_1 \ldots \alpha_k, \Upsilon]\!], \tau \in I(\pi)$
- $(\sigma, 0, \nu) \in [\![\Sigma, \pi\alpha_1 \ldots \alpha_k, \Upsilon]\!]$ iff for some τ such that$(\sigma, \tau, \nu) \in [\![\Sigma, \alpha_1 \ldots \alpha_k, \Upsilon]\!], \tau \notin I(\pi)$

NEGATION:

- $(\sigma, 1, \nu) \in [\![\Sigma, \neg\phi, \Upsilon]\!]$ iff $(\sigma, 0, \nu) \in [\![\Sigma, \phi, \Upsilon]\!]$
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CONJUNCTION:

- $(\sigma, 1, \nu) \in [\![\Sigma, (\phi \wedge \psi), \Upsilon]\!]$ iff $(\sigma, 1, 1, \nu) \in [\![\Sigma, \phi, \psi, \Upsilon]\!]$
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QUANTIFICATION:

- $(\sigma, 1, \nu) \in [\![\Sigma, \forall\alpha\phi, \Upsilon]\!]$ iff $(\sigma, d, 1, \nu) \in [\![\Sigma, \alpha, \phi, \Upsilon]\!]$, for all $d \in D$
- $(\sigma, 0, \nu) \in [\![\Sigma, \forall\alpha\phi, \Upsilon]\!]$ iff $(\sigma, d, 0, \nu) \in [\![\Sigma, \alpha, \phi, \Upsilon]\!]$, for some $d \in D$

$$
\begin{aligned}
1 \in [\![\forall x(Fx \wedge Gx)]\!] \quad & \text{iff} \quad (d, 1) \in [\![x, (Fx \wedge Gx)]\!], \text{for all } d \in D \\
& \text{iff} \quad (d, 1, 1) \in [\![x, Fx, Gx]\!], \text{for all } d \in D \\
& \text{iff} \quad \text{for some } a \text{ such that } (d, a, 1) \in [\![x, x, Gx]\!], \\
& \qquad\quad a \in I(F) \text{ for all } d \in D \\
& \text{iff} \quad \text{for some } a \text{ and some } b \text{ such that } (d, a, b) \in [\![x, x, x]\!], \\
& \qquad\quad a \in I(F) \text{ and } b \in I(G), \text{ for all } d \in D \\
& \text{iff} \quad \text{for all } d \in D, d \in I(F) \text{ and } d \in I(G) \text{ (since } [\![x, x, x]\!] \\
& \qquad\quad = \{(e, e, e) \mid e \in D\})
\end{aligned}
$$

Relationism and Compositionality:

The semantics seems to deliver the results Fine desires.

- $[\![x]\!] = [\![y]\!]$
- $[\![x, y]\!] \neq [\![x, x]\!]$

Enriched Representation:

Every occurrence of the same variable type covaries.It is far less plausible to say that $\forall x Fx \wedge Gx$ ever has the same truth conditions as $\forall x(Fx \wedge Gx)$.

$$
\begin{aligned}
1 \in [\![\exists x Fx \wedge \exists x \neg Fx]\!] \quad & \text{iff} \quad (1,1) \in [\![\exists x Fx, \exists x \neg Fx]\!] \\
& \text{iff} \quad (d_1, 1, 1) \in [\![x, Fx, \exists x \neg Fx]\!], \text{for some } d_1 \in D \\
& \text{iff} \quad (d_1, d_2, 1) \in [\![x, x, \exists x \neg Fx]\!], d_2 \in I(F), \\
& \qquad \text{for some } d_1, d_2 \in D \\
& \text{iff} \quad (d_1, d_2, d_3, 1) \in [\![x, x, x, \neg Fx]\!], d_2 \in I(F), \\
& \qquad \text{for some } d_1, d_2, d_3 \in D \\
& \text{iff} \quad (d_1, d_2, d_3, 0) \in [\![x, x, x, Fx]\!], d_2 \in I(F), \\
& \qquad \text{for some } d_1, d_2, d_3 \in D \\
& \text{iff} \quad (d_1, d_2, d_3, d_4, 0) \in [\![x, x, x, x]\!], d_2 \in I(F) \text{ and} \\
& \qquad d_4 \notin I(F), \text{ for some } d_1, d_2, d_3, d_4 \in D \\
& \text{iff} \quad \text{for some } d \in D, d \in I(F) \text{ and } d \notin I(F) \text{ (since } [\![x, x, x, x]\!] \\
& \qquad = \{(e, e, e, e) \mid e \in D\})
\end{aligned}
$$

Thus, the formula $(\exists x Fx \wedge \exists x \neg Fx)$ cannot assume the value true.

The Antinomy of the Variable

Fine introduces additional semantic inputs: a coordination relation among variables, which one could represent with linking "wires" as follows:

$$\exists x F x \land \exists x \neg F x$$

Fine conceives of the coordination scheme as syntactic in nature, not a semantic parameter.

The Antinomy of the Variable

VARIABLES: $[\![\alpha_1, \ldots, \alpha_n]\!]^c = \{\langle d_1, \ldots, d_n \rangle \in D^n \mid d_i = d_j \text{ iff } c(ij)\}$

ATOMIC:

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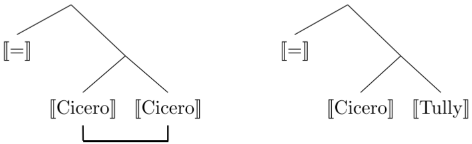
The Antinomy of the Variable

Relationism and Frege' s Puzzle:



" 'Greek' and 'Hellene' are synonymous. But 'All Greeks are Greeks' and 'All Greeks are Hellenes' do not feel quite like synonyms.... The answer is that the logical structure has changed. The first sentence has the form 'All F are F', while the second has the form 'All F are G'."(Putnam, 1954)

The Antinomy of the Variable

Fine tells a story about when coordination happens.

> *When two tokens of a given name are uttered by a single speaker, they will be coordinated if and only if they are internally linked.(Fine, 2007)*

Outline

Semantic Competence and Frege's Puzzle

DIFFERENCE: (1) and (2) differ semantically.

COMPOSITIONALITY: If sentences (1) and (2) differ only by the substitution of constituents which are semantically the same, then (1) and (2) are semantically the same.

MINIMAL PAIR: Sentences (1) and (2) differ only by the substitution of 'Cicero' for 'Tully'–all other inputs to semantic evaluation coincide.

SYNONYMY: 'Cicero' and 'Tully' are semantically the same.

Semantic Competence and Frege's Puzzle

Fine (2007):

- (1a) Cognitive Difference: The two identity sentences are cognitively different;
- (1b) Cognitive Link: If the sentences are cognitively different, then they are semantically different;
- (2) Compositionality: If the sentences are semantically different, then the names "Cicero" and "Tully" are semantically different;
- (3) Referential Link: If the names "Cicero" and "Tully" are semantically different, they are referentially different;
- (4) Referential Identity: The names "Cicero" and "Tully" are not referentially different.

Semantics

Kripke Naming and Necessity (Kripke, 1980):

> *"the four color theorem might turn out to be true and might turn out to be false. It might turn out either way .... the 'might' here is purely 'epistemic'–it merely expresses our present state of ignorance, or uncertainty."*

Semantic Competence and Frege's Puzzle

- (a) Hesperus is Hesperus.
- (b) Hesperus is Phosphorus.

Fregean argument:

- (A) (a) and (b) mean the same.
- ($A \to B$) If (a) and (b) mean the same, then a semantically competent speaker would know that (a) and (b) mean the same.
- ($B \to C$) If a semantically competent speaker would know that (a) and (b) mean the same, then they are equally informative to the speaker.
- ($\neg C$) (a) and (b) differ in informativeness to the competent speaker.
- $\therefore$ Contradiction.

The four premises are jointly inconsistent, the typical textbook choice is to reject the premise (A).

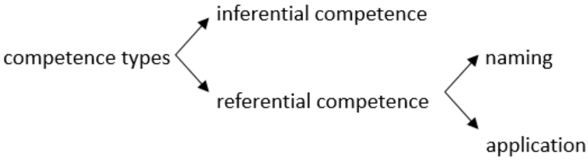
Semantic Competence

Human understanding of natural language:

- Upon understanding a phrase, information is received, knowledge is gained, and qualified decisions are made.
- Often, understanding a sentence is paraphrased as 'grasping it's meaning',
- different strengths of 'grasping the meaning' of the singular term 'my brother'.

A formal theory of semantic competence (to avoid everyday connotations), will be constructed.

Marconi (1997) constructs a conceptual theory of the structure of semantic, lexical competence (SLC).



Each of the three competences correspond to a relation defined over four ontologies.

The four ontologies:

- two external objects, one of external words.
- two mental modules: a word lexicon and a semantic lexicon.

Semantic Competence and Frege's Puzzle

Inferential competence:

- to correctly connect lexical items via the semantic lexicon
- not a matter of logical proficiency and deductive skill
- depends on how well – connected the mental structure of the agent is
- paraphrase, definition, retrieval of a word from its definition, finding a synonym

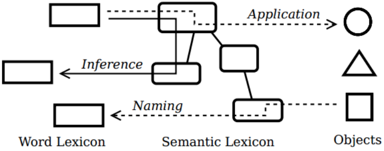
Referential competence – Naming:

- 'what is this called?'
- retrieving a lexical item from the word lexicon when presented with an object.
- two – step process: external object $\longrightarrow$ a suitable concept in the semantic lexicon $\longrightarrow$ a word lexicon item for output.

Application:

- 'hand me the orange'
- identifying an object when presented with a word
- two–stage process: the word lexicon item $\longrightarrow$ a semantic lexicon item $\longrightarrow$ an external object.

Semantic Competence and Frege's Puzzle



You may question:

- why one should distinguish between word and semantic type modules,
- why referential competence is composed of two separate competence types, instead of one bi – directional.

Semantic Competence and Frege's Puzzle

Empirical studies from cognitive neuropsychology indicate that the separation of these systems is mentally real.

The distinction between word lexicon and semantic lexicon:

- patients are able to recognize various objects, but are unable to name them.
- patients are able to reason about objects and their relations when shown objects, but unable to do the same when shown their names.

The latter indicates that reasoning is done with elements from the semantic lexicon, rather than with items from the word lexicon.

Regarding competence types:

- inferential and referential competence are distinct abilities. Specifically, inferential competence and naming are dissociated. One does not imply the other, and *vice verse*.
- application is dissociated from naming, in the sense that application can be preserved while naming is lost.

Modeling the Structure of Lexical Competence

What makes modeling possible?

- the theory has a clearly defined structure
- is based on empirical studies from cognitive neuropsychology.

A two – sorted first – order epistemic logic will be used.

To limitations of space, only the absolutely required elements for the analysis of the argument from the introduction are included.

Two – sorted language:

- ensure that the model respects the dissociation of word lexicon and semantic lexicon.
- σ_{OBJ} is used to represent external objects and the semantic lexicon entries. These are non-linguistic in nature.
- σ_{LEX} is used to represent the lexical items from the agent's language and entries in the word lexicon.

Syntax

Define a language $\mathcal{L}$ with two sorts, σ_{OBJ} and σ_{LEX}.

1. For sort σ_{OBJ}, include:
   - $OBJ = \{a, b, c, \ldots\}$,a countable set of *object constant symbols*
   - $VAR = \{x_1, x_2, \ldots\}$, a countably infinite set of *object variables*

   The set of terms of sort σ_{OBJ} is $TER_{OBJ} = OBJ \cup VAR$.

2. For sort σ_{LEX}, include:
   - $LEX = \{n_1, n_2, \ldots\}$,a countable set of *name constant symbols*
   - $VAR_{LEX} = \{\dot{x}_1, \dot{x}_2, \ldots\}$,a countably infinite set of *name variables*

   The set of terms of sort σ_{LEX} is $TER_{LEX} = LEX \cup VAR_{LEX}$.

3. Include further in $\mathcal{L}$ a unary function symbol, μ, of sort $TER_{LEX} \longrightarrow TER_{OBJ}$.

4. The set of all terms, TER, of $\mathcal{L}$ are $OBJ \cup VAR \cup LEX \cup VAR_{LEX} \cup \{\mu(t)\}$, for all $t \in LEX \cup VAR_{LEX}$.

5. Finally, include the binary relations symbol for identity,$=$.

Syntax

The well-formed formulas of $\mathcal{L}$ are given by

$$\phi ::= (t_1 = t_2)|\neg\phi|\phi \wedge \psi|\forall x\phi|K_i\phi$$

The definitions of the remaining boolean connectives, the dual operator of K_i,$\hat{K}_i$, the existential quantifier and free/bound variables and sentences are all defined as usual. Through a mono-agent system, the operators are indexed by i to allow third-person reference to agent i.

Semantics

Define a model to be a quadruple $M =< W, \sim, Dom, \mathcal{I} >$ where

1. $W = \{w, w_1, w_2, \ldots\}$ is a set of *epistemic alternatives* to actual world w.
2. $\sim$ is an *indistinguishability (equivalence) relation* on $W \times W$.
3. $Dom = Obj \cup Nam$ is the *(constant) domain of quantification*, where $Obj = \{d_1, d_2, \ldots\}$ is a non-empty set of *objects*, and $Nam = \{\dot{n}_1, \dot{n}_2, \ldots, \dot{n}_k\}$ is a finite, non-empty set of *names*.
4. $\mathcal{I}$ is an *interpretation function* such that

$$\mathcal{I} : OBJ \times W \longrightarrow Obj \mid \mathcal{I} : LEX \longrightarrow Nam \mid \mathcal{I} : \{\mu\} \times W \longrightarrow Obj^{Nam}$$

Define a *valuation function*, v, by

$$v : VAR \longrightarrow Obj \mid v : VAR_{LEX} \longrightarrow Nam$$

Semantics

Based on the such models, define the truth conditions for formulas of $\mathcal{L}$ as follows:

$$
\begin{array}{lll}
M, w \models_v (t_1 = t_2) & \text{iff} & d_1 = d_2 \\
 & \text{where} & d_i = \begin{cases} v(t_i) & \text{if } t_i \in VAR \cup VAR_{LEX} \\ \mathcal{I}(w, t_i) & \text{if } t_i \in OBJ \\ \mathcal{I}(t_i) & \text{if } t_i \in LEX \end{cases} \\
M, w \models_v \phi \wedge \psi & \text{iff} & M, w \models_v \phi \text{ and } M, w \models_v \psi \\
M, w \models_v \neg\phi & \text{iff} & \text{not } M, w \models_v \phi \\
M, w \models_v K_i\phi & \text{iff} & \text{for all } w' \text{ such that } w \sim w', M, w' \models_v \phi \\
M, w \models_v \forall x \phi(x) & \text{iff} & \text{for all } x\text{-variants } v' \text{ of } v, M, w \models_{v'} \phi(x)
\end{array}
$$

$QS5(\sigma_{LEX}, \sigma_{OBJ})$ represent the structure and properties of the SLC in two steps:

1. represent the ontologies of the SLC
2. the model can express the three competence types and the dissociation properties are preserved

The external objects constitute the sub-domain *Obj*, and are denoted in the syntax by the terms TER_{OBJ}.

External words (proper names) constitute the sub-domain *Nam* denoted by the terms TER_{LEX}.

In order to define the semantic lexion, first define an *object indistinguishability relation* $\sim_w^a$:

$$d \sim_w^a d' \text{ iff } \exists w' \sim w \ : \ \mathcal{I}(a,w) = d \text{ and } \mathcal{I}(a,w') = d',$$

and from this define the agent's *individual concept class for a at w by*

$$C_w^a(d) = \{d' \mid d \sim_w^a d'\}.$$

The semantic lexicon of agent *i*:

$$SL_i = \{C_w^a(d) \ : \ C_w^a(d) \neq \emptyset\}$$

Semantic Competence and Frege's Puzzle

The word lexicon case is simpler, because the name constants are rigid.

$$(n_1 = n_2) \to K_i(n_1 = n_2)$$

$$K_i(n_1 = n_2) \to \exists \dot{x} K_i(\dot{x} = n_2)$$

Due to the simpler definition of $\mathcal{I}$ for name constants, we can define i's *name class for n* directly. Where $\dot{n} \in Nam$ and $n \in LEX$ this is the set $C_i^n(\dot{n}) = \{\dot{n}' : \mathcal{I}(n) = \dot{n}'\}$. The word lexicon of i is then the collection of such sets: $WL_i = \{C_i^n(\dot{n}) : n \in LEX\}$.

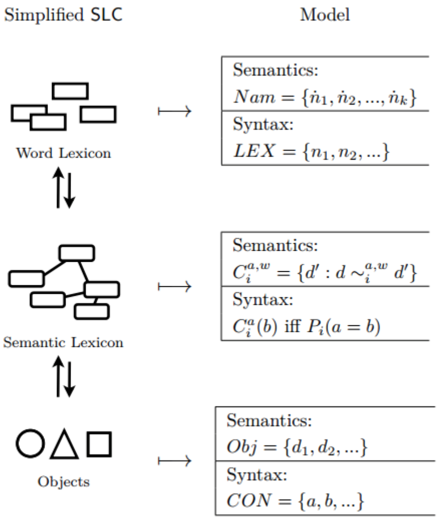


Figure: (Rendsvig, 2011)

Semantic Competence and Frege's Puzzle

Inferential Competence:

- Only proper name, only identity relation. Knowledge 'a is P' cannot be expressed。
- **(Knowledge of Co-reference)**. Agent *i knows that names n and n′ co-refer* in pointed model (M, w) iff

$$M, w \models_v K_i(\mu(n) = \mu(n'))$$

- **(Full Inferential Competence)**. Agent *i* is fully inferentially competent with respect to *n* in pointed model (M, w) iff

$$M, w \models_v (\mu(n) = \mu(n')) \rightarrow K_i(\mu(n) = \mu(n'))$$

for all $n' \in LEX$.

Semantic Competence and Frege's Puzzle

Referential competence has two distinct relations, *application and naming*, between names and objects, relating these through the semantic lexicon.

Application:

- identify the appropriate referent, when presented with a name n, i.e. use or apply a name.
- this ability can be expressed of the agent with respect to name n in w by

$$M, w \models_v \exists x K_i(\mu(n) = x)$$

- given the assumption of syntactical competence, there is no uncertainty regarding which name is presented.

Naming:

- produce a correct name when presented with an object, say a,
- the de re formula $\exists \dot{x} K_i(\mu(\dot{x}) = a)$ is insufficient, $\mu(\dot{x})$ and a may simply co-vary across states.
- naming must include a requirement that i can identify a, and know a name for a. This is:

$$M, w \models_v \exists x \exists \dot{x} K_i((x = a) \wedge (\mu(\dot{x}) = a))$$

.

Semantic Competence and Frege's Puzzle

Dissociations, for example, application does not imply naming:

$$w_1 \models_v (\mu(n) = a) \wedge \exists x K_i(\mu(n) = x),$$

$$\text{but } w_1 \models_v \neg\exists x \exists \dot{x} K_i((x = a) \wedge (\mu(\dot{x}) = a)).$$

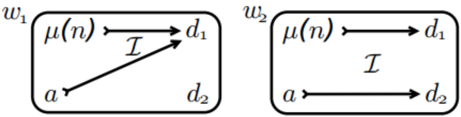


Figure: A simplified illustration of the SLC.(Lassiter and Slavkovik, 2012)

Semantic Competence and Frege's Puzzle

Evaluate this argument in the formal setting:

- assume (A) is satisfied at actual word w in a model M

$$(\mu(n) = \mu(n)) \leftrightarrow (\mu(n) = (n')) \qquad \text{(A*)}$$

- Since the left-hand identity is a validity, (A) amounts to that the actual world w in model M satisfies

$$(\mu(n) = \mu(n'))$$

- The second premise is that (A*) implies that any competent speaker knows that

$$(\mu(n) = \mu(n)) \leftrightarrow (\mu(n) = \mu(n')).$$

  Its truth depends on the type of competence meant.
- The last three premises of the argument will be run through using inferential competence and application.
- The ability to name objects is not relevant for the present.

Semantic Competence and Frege's Puzzle

$$(A) \qquad M, w \models_v \mu(n) = \mu(n')$$

$$\Downarrow$$

$$(B_1) \qquad M, w \models_v K_i(\mu(n) = \mu(n'))$$

$$\Downarrow$$

$$(C) \qquad \neg\exists w' \sim_i w : M, w' \models_v \neg(\mu(n) = \mu(n'))$$

$$(\neg C) \qquad \exists w' \sim_i w : M, w' \models_v \neg(\mu(n) = \mu(n'))$$

$(\neg C)$ is false as a consequence of the assumption of Millianism and agent i's inferential competence with respect to n and n'.

The inferential competence of agent i is constituted by i's ability to find synonyms when prompted with names. As this is a knowledge-based ability, the knowledge that the identity statement is supposed to provide is already assumed to be possessed by the agent.

Semantic Competence and Frege's Puzzle

Referential Competence: Application

(A) $M, w \models_v \mu(n) = \mu(n')$

$\Downarrow$?

(B$_2$) $M, w \models_v K_i(\mu(n) = \mu(n'))$

$\Downarrow$

(C) $\neg\exists w' \sim_i w : M, w' \models_v \neg(\mu(n) = \mu(n'))$

($\neg$C) $\exists w' \sim_i w : M, w' \models_v \neg(\mu(n) = \mu(n'))$

(A $\rightarrow$ B$_2$): $(\mu(n) = \mu(n') \wedge \exists x K_i(\mu(n) = x) \wedge \exists y K_i(\mu(n') = y)) \rightarrow K_i(\mu(n) = \mu(n'))$

Semantic Competence and Frege's Puzzle

定义 (Context Structure)

Where W is a non-empty set of worlds, a *context structure on* W is a pair (S, Act) where S is a partition of W

$$S = \{S_1, S_2, \ldots, S_n\}$$

where each S_k contains an actual world w_k from the set of *actual worlds*,

$$Act = \{w_1, w_2, \ldots, w_3\}$$

定义 (Context Distinguishability)

Where S_k is a context from (S, Act), an indistinguishability relation $\sim_i$ that *distinguishes contexts* satisfy

$$\text{If} w \sim_i w' \text{ then } w, w' \in S_k.$$

Define the set of agent *i's epistemic alternatives to* w_k by

$$S_k^i = \{w : (w_k, w) \in \sim_i\}$$

Semantic Competence and Frege's Puzzle

定义 (Objective Possibility)

Where *Act* is the set of actual worlds from (S, Act), define the *objective possibility relation* by

$$R = Act \times Act.$$

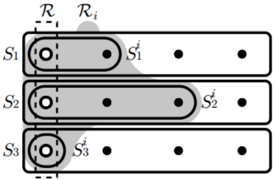


Figure: (Rendsvig, 2011)

Semantic Competence and Frege's Puzzle

定义 (Subjective Possibility)

Where $S = S_1, \ldots, S_n$ is the partition from (S, Act), the *subjective possibility relation for agent i* is defined by

$$R_i = \bigcup_{k \leq n} S_k^i \times \bigcup_{k \leq n} S_k^i$$

定义 (Context Model)

A *context model M* is a tuple

$$M = \langle W, (S, Act), (\sim_i, R_i)_{i \in I}, R, Dom, \mathcal{I} \rangle$$

Semantic Competence and Frege's Puzzle

Syntax:

$$\Box\phi \mid \Box_i\phi$$

Semantics:

$$M, w \models_v \Box\phi \quad \text{iff} \quad \forall w \in Act, M, w \models_v \phi$$
$$M, w \models_v \Box_i\phi \quad \text{iff} \quad \forall w' : wR_iw', \Rightarrow M, w' \, models_v \phi$$

The reading of the box operators are 'in all contexts, ϕ' and 'in all contexts, for all *i* knows, ϕ'.

Semantic Competence and Frege's Puzzle

it is required of that

$$\forall w, w' \in Act : M, w \models_v (\mu(n) = a) \Rightarrow M, w' \models_v (\mu(n) = a)$$

$$(\mu(n) = \mu(n')) \rightarrow \Box(\mu(n) = \mu(n'))$$

i.e. that co-reference of names is objectively necessary.

physical identity statements persistence across actual worlds, will be assumed:

$$\forall w, w' \in Act : M, w \models_v (a = b) \Rightarrow M, w' \models_v (a = b)$$

Semantic Competence and Frege's Puzzle

S_k Inferential Competence

定义 (S_k Knowledge of Co-reference)

Where w_k is the actual world of context S_k from model M, agent i is said to have S_k *knowledge of co-reference of n and* n_0 iff

$$M, w_k \models_v K_i(\mu(n) = \mu(n'))$$

Universally have knowledge of coreference:

$$\Box K_i(\mu(n) = \mu(n'))$$

By definition,

$$\Box_i(\mu(n) = \mu(n'))$$

It *does not* imply

$$\exists x \Box_i(\mu(n) = x)$$

Semantic Competence and Frege's Puzzle

S_k Application:

$$M, w_k \models_v \exists x K_i(\mu(n) = x)$$

universal application:

$$\Box \exists x K_i(\mu(n) = x)$$

Semantic Competence and Frege's Puzzle

$$M, w_k \models_v (\mu(n_p) = v) \wedge (\mu(n_h) = v)$$

$$M, w_k \models_v (h = v) \wedge (p = v)$$

$$M, w_1 \models_v (\exists x K_i(p = x) \wedge (\mu(n_p) = x)$$

$$M, w_2 \models_v (\exists x K_i(h = x) \wedge (\mu(n_h) = x)$$

$$M, w_k \models_v \neg K_i(\mu(n_p = \mu(n_h)))$$

A model can still be constructed satisfying all the above assumptions.

Semantic Competence and Frege's Puzzle

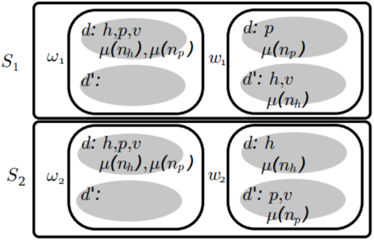


Figure: (Rendsvig, 2011)

Hence an assumption of lacking inferential competence does not result in a contradiction.

Outline

Conclusion

- Frege's Puzzle:
  - $a = a$ vs. $a = b$
  - Meaning consists of sense and reference
- Kit Fine:
  - The Antinomy of the variable
  - Relationism and Frege′ s Puzzle
  - coordination?
- Semantic Competence and Frege's Puzzle
  - Cognitive link and Fregean argument
  - semantic competence: SLC
  - inferential competence and applying

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